A NUMERICAL COMPUTATION OF WATER QUALITY MEASUREMENT IN A UNIFORM CHANNEL USING A FINITE DIFFERENCE METHOD

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บทคัดย่อ

งานวิจัยนี้มีจุดมุ่งหมายเพื่อประยุกต์ตัวแบบเชิงคณิตศาสตร์ซึ่งเรียกว่า สมการการพา-การแพร่

$$-D_x \frac{d^2C}{dx^2} + u\frac{dC}{dx} + RC - Q = 0$$

และวิธีผลต่างอันตะ

$$y'' = p(x)y' + q(x)y + r(x)$$

กับปัญหาการประมาณระดับมลพิษในลำน้ำเอกรูป โดยจะทำการปรับเปลี่ยนตัวแปรเสริมบางตัวของระเบียบวิธีข้างต้น และเพื่อสร้างตัวแบบออพติไมเซชันแบบมีเงื่อนไขในการควบคุมระดับมลพิษและค่าใช้จ่ายให้อยู่ภายใต้กรอบของกฎหมายและงบประมาณของโรงงาน โดยทั่วไประดับค่ามลพิษจะได้มาจากการเก็บข้อมูลภาคสนาม ซึ่งมักเกิดความยุ่งยากและมีโอกาสคลาดเคลื่อนได้ในแต่ละตำแหน่งที่ทำการเก็บข้อมูล

กระบวนการการดำเนินงานวิจัยเริ่มจากการใช้วิธีผลต่างอันตะเพื่อหาผลเฉลยเชิงตัวเลขของสมการเชิงอนุพันธ์สามัญเชิงเส้นอันดับสองที่มีเงื่อนไขขอบแบบดีรีเคล และแบบนอยมันน์ วิธีผลต่างอันตะใหม่ที่ได้จะใช้เพื่อหาผลเฉลยเชิงตัวเลขของสมการการพา-การแพร่ สำหรับการประมาณระดับความเข้มข้นของมลพิษในลำน้ำเอกรูป ผลเฉลยดังกล่าวจะอยู่ในรูปของสมการเชิงเส้น ต่อจากนั้นจะนำระบบสมการเชิงเส้นนี้ไปสร้างเป็นตัวแบบออพติไมเซชัน ซึ่งฟังก์ชันวัตถุประสงค์ของตัวแบบคือเพื่อทำให้ค่าใช้จ่ายในการควบคุมระดับมลพิษน้อยที่สุด และข้อจำกัดของตัวแบบจะรวมถึงข้อกำหนดทางกฎหมายที่เกี่ยวข้อง และงบประมาณที่ให้กับการควบคุมมลพิษของโรงงาน

ผลที่ได้รับจากงานวิจัยแสดงว่า สมการการพา-การแพร่และวิธีผลต่างอันตะที่ปรับเปลี่ยนแล้วมีความเหมาะสมสำหรับตรวจวัดระดับมลพิษในลำน้ำเอกรูป และผลเฉลยเชิงตัวเลข

$$(-2)w_{N-1}+(2+h^2q(x_N)w_N)=-h^2r(x_N)+(2-hp(x_N))h\beta$$

นำมาสร้างเป็นตัวแบบออพติไมเซชัน ช่วยให้โรงงานควบคุมค่าใช้จ่ายในการบำบัดน้ำเสีย และยังคงรักษาระดับมลพิษให้เป็นไปตามที่กฎหมายกำหนด เทคนิคและวิธีการที่นำเสนอนี้สามารถถูกนำไปประยุกต์ใช้กับปัญหาการควบคุมมลพิษการตรวจวัดคุณภาพน้ำในลำน้ำแบบเอกรูปของโรงงานอื่นๆ ได้

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ABSTRACT

The purposes of this research are to apply a mathematical model called the convection-diffusion equation

$$-D_x \frac{d^2C}{dx^2} + u\frac{dC}{dx} + RC - Q = 0,$$

and a finite difference method

$$y'' = p(x)y' + q(x)y + r(x),$$

to the water pollution approximation problem in a uniform channel by modifying some parameters of the above methods and to formulate a constrained optimization model to keep the pollution levels and the associated budgets within acceptable ranges. In general, the pollution levels are measured via field data collection which is often complicated and erroneous at some data sources.

The processes of this research begin by using a finite difference method to find a numerical solution of a second-order linear ordinary differential equation with Dirichlet and Neumann boundary conditions. Then a new finite difference method is proposed to solve the convection-diffusion equation for approximating the water pollution concentration levels in the uniform channel. The obtained solutions are in the form of linear equations. And then, these solutions are used to formulate an optimization model, of which the objective function is to minimize the costs of water treatment and the constraints include the legal regulations and the planned budgets of the factory.

The results of this research show that the modified convection-diffusion equation and the finite difference method are suitable to water pollution level approximation in a uniform channel

and the numerical solution

$$(-2)w_{N-1} + (2 + h^2 q(x_N) w_N) = -h^2 r(x_N) + (2 - hp(x_N))h\beta,$$

used to formulate the optimization model, assists the factory in controlling the expenses of water treatment while the pollution levels are in the legal regulations. The proposed techniques and methods can also be applied to the water pollution approximation problem in a uniform channel of other factories.

Keywords : Numerical computation, Water quality measurement, Uniform channel, Convection-diffusion equation, Optimization

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List of Abbreviations

kg/m^3	Kilogram per cubic metre
Kg/m^3s	Kilogram per cubic metre second
km	Kilometre
m	Metre
mg/l	Milligram per liter
m^2/s	Square metre per second
m/s	Metre per second
s^{-1}	Per second

CHAPTER 1

INTRODUCTION

A Mathematical Model of Water Pollution Using Finite Element Method (N. Pochai and S. Tangmanee, 2007). The finite element method for assessment of the COD concentration in a river, lake or another closed water area is considered. This model requires the permanent current and substance dispersion patterns. The chemical oxygen demand (COD) test is commonly used to indirectly measure the amount of organic compounds in water. COD is used to determine the amount of organic pollutants surface water. It is expressed in milligrams per liter (mg/l), which indicated the mass of oxygen consumed per liter of solution. The mathematical model for solving the dispersion of pollutant in a river. A finite difference method for assessment of the chemical oxygen demand (COD) concentration in a river is considered. The finite element method is used for water quality measurement and control in one-dimensional and two-dimensional domains. This model requires the calculation of the substance dispersion given the water velocity in the channel. In this topic, a finite difference method is used to compute the concentration of the pollutant for variable inputs. A numerical example is also given.

1.1 Background and Statement of the Problem

The increase in an industrial occupation is the principal reason for the growth of pollution. Water quality must be protected and maintained for several uses, the principal ones being domestic water supply, energy production, industry, agriculture, fish and wildlife. The highest priority use is domestic water supply, with priorities for other uses depending largely on local or regional conditions and factors. Water pollution can effect humans in many ways, depending on the purpose for which the water resources are to be used. Since it affects human lives, it is health problem.

The term to pollute may be defined as to destroy the purity of or to make foul or dirty. Water pollution may therefore be defined as the alteration of the characteristics of a receiving water body in such a way as to make it unfit for one or more specific uses. To state it another way, pollution refers to the changes in the natural physical, chemical, and biological characteristics of a receiving water caused by the discharge of any material into that water that detracts from beneficial use.

Control of pollution is necessary for the protection of the water environment and the maintenance of acceptable quality in rivers, lakes, reservoirs, streams, estuaries, oceans, and groundwater. The standard, in turn, will depend on the uses to be made of the receiving waters: water supply, fishing-wildlife, industrial, and other uses.

The methods to detect the amount of pollutant both in the air and water mostly are conducted by a field measurement and a mathematical simulation.

1.2 Purpose of the Study

To apply the one-dimensional of the convection diffusion equations to the water pollution problem in the channel with contaminant discharged. The convection and diffusion of the pollutant, the concentration of pollutant at any point in the domain will be approximated by the finite difference method. The steady state flow will be considered. The velocity of the current will be formulated as a known velocity function.

Optimal control for the minimum cost of water treatment will be formulated and discussed. Computer program for working out the approximate model will be constructed.

1.3 Theoretical Perspective

Theoretical perspective of the thesis is restricted to the application of the finite difference method to the water pollution problem in the stream, measurement and control, in the case of steady flow with regular boundaries.

1.4 Delimitations and Limitations of the Study

In this thesis describes the mathematical modelling of the water pollution measurement and control in the water area. We have to establish a simulation process by means of which water pollution levels can be reduced to an agreed standard at the lowest cost. The first part gives the detail of the basic knowledge of the mathematical modelling for water pollution measurement in one-dimensional problem.

The second part gives the computation of the steady water quality measurement involved the numerical solution of a convection-diffusion equation. We also gives the fundamental of the finite difference formulation that the numerical methods to approximate the pollutant concentration in the one-dimensional water areas such as stream.

Finally, we give the method of water pollution control. In this part, the finite difference method for solving the one-dimensional steady convection-diffusion equation with variable coefficients is presented; it is then used to optimize water treatment costs.

1.5 Significance of the Study

The definition of terms of this thesis are the applicable model of water pollution assessment and cost optimal control of water pollution treatment in the stream.

CHAPTER 2

BASIC KNOWLEDGE

2.1 Literature Review

In 2007, Pochai, N. and Tangmanee, S. give a mathematical model of water pollution using finite element method. The finite element method for solving the one-dimension and two-dimension of steady state convection-diffusion equation with constant coefficients of nearly closed water area is presented.

In 2006, Pochai, N., Tangmanee, S., Crane L.J and Miller, J.J.H propose a mathematical model of water pollution control using the finite element method. They give the one-dimensional steady convection-diffusion equation with constant coefficients. It is then used to optimize water treatment costs.

MacDonald, 1995 showed Linear Programming using Microsoft Excel Solver.

2.2 Water Quality Model

In order to calculate the solutions of mathematical model of water pollution, we requires some basic knowledge of the flow analysis. In this chapter, we will give the detail of some preliminaries of basic flow properties. We will also present the basic flow equation and convection-diffusion equation with associated boundary conditions for one-dimensional system.

2.2.1 Governing equation : Convection – Diffusion equation

This section gives a physical meaning of a mathematical model to the water pollutant dispersion in river. The dispersion of the COD concentration is described by the convection – diffusion equation on COD in an arbitrary domain $\Omega \subseteq \Re^n$, n = 1, 2, 3

$$\frac{\partial \Gamma}{\partial t} + \Upsilon \nabla \Gamma = D \nabla^2 \Gamma , \qquad (2.1)$$

where $\Gamma(x,t)$ is the concentration fields, $\Upsilon(x,t)$ is the velocity fields, and D is the molecular diffusivity. Both Γ and Υ are function of position x and time t . The domain boundary can be of two types are Ω_1 with specified COD concentration and Ω_2 with specified flux of concentration, and total boundary $\partial\Omega$ is $\Omega_1 \cup \Omega_2 = \phi$.

The boundary condition on Ω_1 and Ω_2 are

$$C = C_0 \quad \text{on } \Omega_1, \tag{2.2}$$

$$\frac{\partial C}{\partial n} = T_0 \quad \text{on } \Omega_2. \tag{2.3}$$

2.2.2 One dimensional convection – diffusion equation

A mathematical model for described the dispersion of COD concentration in one – dimensional problem, eg. river, channel, uniform channel and etc. (see Fig 2.1) will be presented. In the case of one – dimensional problem, we can deduce these problem to be the steady state convection – diffusion equation in an interval domain [a,b]

$$-D_x \frac{d^2 C}{dx^2} + u\frac{dC}{dx} + RC - Q = 0, \tag{2.4}$$

where

$C(x)$: concentration of COD at the point $x \in [a,b]$ (kg/m^3),

u : flow velocity in x directions (m/s),

D_x : diffusion coefficient (m^2/s),

R : the substance decay rate (s^{-1}),

Q : the increasing rate of substance concentration due to a source $(Kg/m^3 s)$.

The boundary conditions are

$$C = C_0 \text{ at } x = a \quad \text{and} \quad \frac{dC}{dx} = T_0 \text{ at } x = b.$$

Figure 2.1 Uniform Channel

(Khlong Lad Poe, Samut Prakarn,Thailand,http://www.nationgroup.com)

2.3 Numerical schemes

2.3.1 Finite difference method for second order linear ordinary differential equation

The finite difference methods in this section have good agreement stability characteristics, but they generally require more work to obtain a specified accuracy. Methods involving finite differences for solving boundary-value problems replace each of the derivatives in the differential equation by an appropriate difference-quotient approximation. The difference quotient is chosen to maintain a specified order of truncation error. The linear second-order boundary-value problem,

$$y'' = p(x)y' + q(x)y + r(x), \qquad a \le x \le b, \qquad y(a) = \alpha, \qquad y(b) = \beta \tag{2.5}$$

requires that difference-quotient approximations be used to approximate both y' and y''. First, we select an integer $N > 0$ and divide the interval $[a,b]$ into $(N+1)$ equal subintervals, whose endpoints are the mesh points $x_i = a + ih,$ for $i = 0,1,\ldots,N+1,$ where $h = (b-a)/(N+1)$. Choosing the constant h in this matter facilitates the application of a matrix algorithm, which solves a linear system involving an $N \times N$ matrix.

2.3.1.1 A centered-difference formula for y''

At the interior mesh points, x_i, for $i = 1,2,\ldots,N$, the differential equation to be approximated is

$$y''(x_i) = p(x_i)y'(x_i) + q(x_i)y(x_i) + r(x_i). \tag{2.6}$$

Expanding y in a third Taylor polynomial about x_i evaluated at x_{i+1} and x_{i-1}

$$y_{i+1} = y_i + y'(x_i)h + \frac{y''(x_i)h^2}{2!} + \frac{y'''(x_i)h^3}{3!} + \ldots + \frac{y^{(k)}(x_i)h^k}{k!} + R . \tag{2.7}$$

We have

$$y(x_{i+1}) = y(x_i + h) = y(x_i) + hy'(x_i) + \frac{h^2}{2} y''(x_i) + \frac{h^3}{6} y'''(x_i) + \frac{h^4}{24} y^{(4)}(\xi_i^+), \tag{2.8}$$

for some $\xi_i^+ \in (x_i, x_{i+1})$, and

$$y(x_{i-1}) = y(x_i - h) = y(x_i) - hy'(x_i) + \frac{h^2}{2} y''(x_i) - \frac{h^3}{6} y'''(x_i) + \frac{h^4}{24} y^{(4)}(\xi_i^-), \tag{2.9}$$

for some $\xi_i^- \in (x_{i-1}, x_i)$, assuming $y \in C^4[x_{i-1}, x_{i+1}]$. If these equations are added, the terms involving $y'(x_i)$ and $y'''(x_i)$ are eliminated, and simple algebraic manipulation gives

$$y''(x_i) = \frac{1}{h^2}[y(x_{i+1}) - 2y(x_i) + y(x_{i-1})] - \frac{h^2}{24}[y^{(4)}(\xi_i^+) + y^{(4)}(\xi_i^-)]. \tag{2.10}$$

The Intermediate Value Theorem can be used to simplify this even further:

$$y''(x_i) = \frac{1}{h^2}[y(x_{i+1}) - 2y(x_i) + y(x_{i-1})] - \frac{h^2}{12} y^{(4)}(\xi), \tag{2.11}$$

for some ξ in (x_{i-1}, x_{i+1}). This is called the **centered-difference formula** for $y''(x_i)$.

2.3.1.2 A centered-difference formula for y'

A centered-difference formula for $y'(x_i)$ is obtained in a similar manner,

$$y_{i+1} = y_i + y'(x_i)h + \frac{y''(x_i)h^2}{2!} + \frac{y'''(x_i)h^3}{3!} + \ldots + \frac{y^{(k)}(x_i)h^k}{k!} + R .$$

We have

$$y(x_{i+1}) = y(x_i + h) = y(x_i) + hy'(x_i) + \frac{h^2}{2} y''(x_i) + \frac{h^3}{6} y'''(x_i) + \frac{h^4}{24} y^{(4)}(\xi_i^+),$$

for some $\xi_i^+ \in (x_i, x_{i+1})$, and

$$y(x_{i-1}) = y(x_i - h) = y(x_i) - hy'(x_i) + \frac{h^2}{2} y''(x_i) - \frac{h^3}{6} y'''(x_i) + \frac{h^4}{24} y^{(4)}(\xi_i^-),$$

for some $\xi_i^- \in (x_{i-1}, x_i)$, assuming $y \in C^4[x_{i-1}, x_{i+1}]$. If these equations are minused, the terms involving $y''(x_i)$ and $y^{(4)}(\xi_i)$ are eliminated, it follows that

$$y'(x_i) = \frac{1}{2h}[y(x_{i+1}) - y(x_{i-1})] - \frac{h^2}{6} y'''(\eta_i), \text{ for some } \eta_i \text{ in } (x_{i-1}, x_{i+1}). \tag{2.12}$$

2.3.1.3 A Matrix Form of Solution

The use of these centered-difference formulas in Eq. (2.6) results in the equation

$$y''(x_i) = p(x_i)y'(x_i) + q(x_i)y(x_i) + r(x_i),$$

$$\frac{1}{h^2}\left[y(x_{i+1})-2y(x_i)+y(x_{i-1})\right]-\frac{h^2}{12}y^{(4)}(\xi_i)=p(x_i)\left\{\frac{1}{2h}\left[y(x_{i+1})-y(x_{i-1})\right]-\frac{h^2}{6}y'''(\eta_i)\right\}$$
$$+q(x_i)y(x_i)+r(x_i). \tag{2.13}$$

We have,

$$\frac{1}{h^2}\left[y(x_{i+1})-2y(x_i)+y(x_{i-1})\right]=p(x_i)\left[\frac{y(x_{i+1})-y(x_{i-1})}{2h}\right]+q(x_i)y(x_i)+r(x_i)$$
$$-\frac{h^2}{12}\left[2p(x_i)y'''(\eta_i)-y^{(4)}(\xi_i)\right]. \tag{2.14}$$

A finite difference method with truncation error of order $O(h^2)$ results by using Eq. (2.14) together with the boundary conditions $y(a)=\alpha$ and $y(b)=\beta$, we define

$$w_0=\alpha \quad \text{and} \quad w_{N+1}=\beta, \tag{2.15}$$

we can obtain

$$\frac{1}{h^2}\left[-y(x_{i+1})+2y(x_i)-y(x_{i-1})\right]+p(x_i)\left[\frac{y(x_{i+1})-y(x_{i-1})}{2h}\right]+q(x_i)y(x_i)=-r(x_i), \tag{2.16}$$

$$\left(\frac{2w_i-w_{i+1}-w_{i-1}}{h^2}\right)+p(x_i)\left(\frac{w_{i+1}-w_{i-1}}{2h}\right)+q(x_i)w_i=-r(x_i), \tag{2.17}$$

for each $i=1,2,\ldots,N$, Eq. (2.17) can be obtained

$$-\left(1+\frac{h}{2}p(x_i)\right)w_{i-1}+\left(2+h^2q(x_i)\right)w_i-\left(1-\frac{h}{2}p(x_i)\right)w_{i+1}=-h^2r(x_i), \tag{2.18}$$

and the resulting system of equations is expressed in the tridiagonal $N\times N$ -matrix form

$$AW=B, \tag{2.19}$$

$$A = \begin{bmatrix} 2+h^2q(x_1) & -1+\frac{h}{2}p(x_1) & & & \\ -1-\frac{h}{2}p(x_2) & 2+h^2q(x_2) & -1+\frac{h}{2}p(x_2) & & \\ & \cdots & \cdots & & \\ & & \cdots & \cdots & \\ & & & & -1+\frac{h}{2}p(x_{N+1}) \\ & & & -1-\frac{h}{2}p(x_N) & 2+h^2q(x_N) \end{bmatrix},$$

$$W = \begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-1} \\ w_N \end{Bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -h^2r(x_1)+\left(1+\frac{h}{2}p(x_1)\right)w_0 \\ -h^2r(x_2) \\ \vdots \\ -h^2r(x_{N-1}) \\ -h^2r(x_N)+\left(1-\frac{h}{2}p(x_N)\right)w_{N+1} \end{bmatrix}.$$

2.3.2 Finite difference methods for second order linear ordinary differential equation with Robin boundary conditions

We considered the one dimensional two-point boundary value problem from Eq. (2.5)

$$y'' = p(x)y' + q(x)y + r(x), \quad x \in [a,b].$$

Subject to the Dirichlet boundary conditions

$$y(a) = \alpha \text{ and } y(b) = \beta.$$

The pollutant concentration at each end of the channel might not be known. For example, we might only know that the end at $x = b$ was steady, so that there was no flux from that end. This would give rise to a boundary condition of the form

$$y'(b) = 0. \tag{2.20}$$

A boundary condition of this form, in which the value of the derivative is specified, is known as a Neumann boundary condition. We also might only know that discharge pollutant concentration is taking place at the end $x = a$. This would give rise to the boundary condition

$$y(a) = C_0. \tag{2.21}$$

A boundary condition of this type, in which a linear combination of the value of the function and the value of the first derivative is specified, is known as a Robin boundary condition.

We will investigate the formulation of finite difference approximations for linear boundary value problems subject to both Neumann and Robin boundary conditions.

2.3.2.1 Non-Dirichlet Boundary Conditions

Because the general Neumann boundary condition

$$y'(a)=\alpha \quad \text{or} \quad y'(b)=\beta, \tag{2.22}$$

is just a special case of the general Robin boundary condition

$$\alpha_1 y(a)+\alpha_2 y'(a)=\alpha_3 \quad \text{or} \quad \beta_1 y(b)+\beta_2 y'(b)=\beta_3. \tag{2.23}$$

Setting $\alpha_1 = 0$ or $\beta_1 = 0$, we will develop the system of algebraic equations for the finite difference approximation to the linear boundary value problem

$$y'' = p(x)y' + q(x)y + r(x), \quad x \in [a,b], \tag{2.24}$$

subject to the Robin boundary conditions

$$\alpha_1 y(a)+\alpha_2 y'(a)=\alpha_3, \tag{2.25}$$

$$\beta_1 y(b)+\beta_2 y'(b)=\beta_3. \tag{2.26}$$

We will assume that $\alpha_2 \neq 0$ and $\beta_2 \neq 0$. For the computational grid, let N be a positive integer, and partition the interval $[a,b]$ into

$$a = x_0 < x_1 < x_2 < \cdots < x_{N-1} < x_N = b, \tag{2.27}$$

where $x_i = a + ih$ and $h = (b-a)/N$. Further, let w_i denote the approximation to the exact solution, $y(x)$, at $x = x_i$. We need $N+1$ equations to determine the values $w_0, w_1, w_2, \ldots, w_N$. These are $N-1$ of these equations are obtained as in the previous section: Evaluate the differential equation at each interior grid point $x = x_i (1 \le i \le N-1)$, replace the derivatives by second-order central difference approximations, drop the truncation error terms and collect like terms. The resulting computational template is

$$\left(-1-\frac{h}{2}p_i\right)w_{i-1} + \left(2+h^2 q_i\right)w_i + \left(-1+\frac{h}{2}p_i\right)w_{i+1} = -h^2 r_i. \tag{2.28}$$

Consider Eq. (2.25) at $x_0 = a$ and $x_N = b$,

$$\alpha_1 y(a)+\alpha_2 y'(a)=\alpha_3.$$

To maintain the first-order accuracy of the equations, we could replace the derivative in the boundary condition by the $O(h)$ central difference approximation, Eq. (2.12) leads to

$$y_i' = \frac{y(x_{i+1}) - y(x_{i-1})}{2h}, \tag{2.29}$$

The coefficient matrix and the second-order accuracy of the approximation, is to introduce a "fictitious node" to the computational grid. Applying the computational template for the differential equation at $x = x_0$ produces

$$\left(-1-\frac{h}{2}p_0\right)w_f + \left(2+h^2 q_0\right)w_0 + \left(-1+\frac{h}{2}p_0\right)w_1 = -h^2 r_0, \tag{2.30}$$

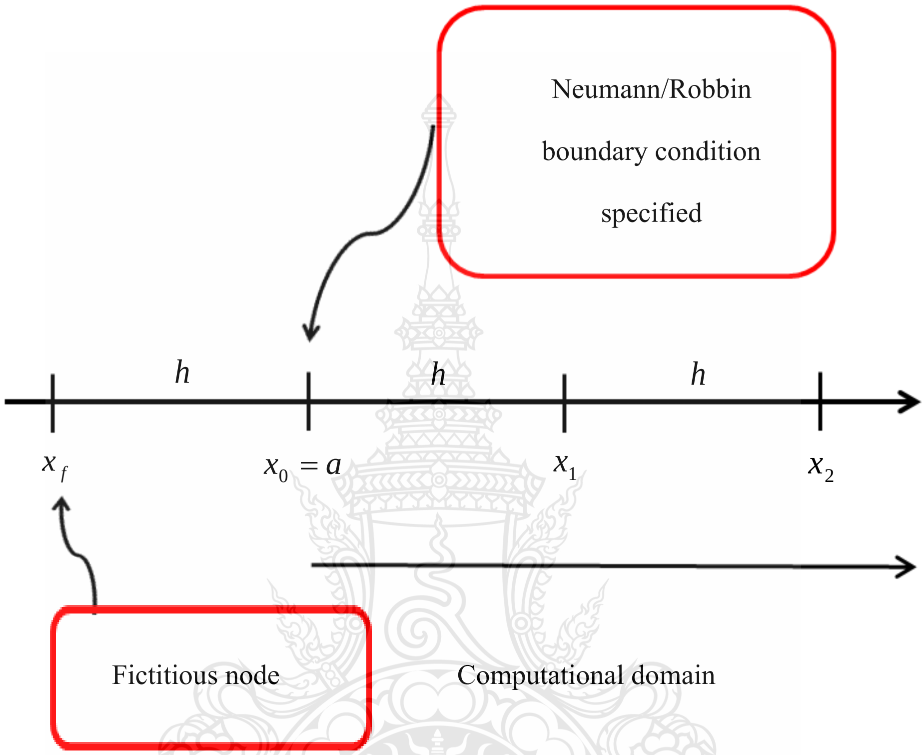

Figure 2.2 Fictitious node

of course, w_f must be eliminated from this equation. By applying the Robin boundary condition of Eq. (2.25),

$$\alpha_1 y(a) + \alpha_2 y'(a) = \alpha_3,$$

we obtain

$$\alpha_1 w_0 + \alpha_2 \frac{w_1 - w_f}{2h} = \alpha_3, \tag{2.31}$$

where we have replaced the first derivative with its second-order central difference approximation. Solving w_f yields

$$w_f = w_1 - \frac{2h}{\alpha_2}\left(\alpha_3 - \alpha_1 w_0\right). \tag{2.32}$$

Substituting Eq. (2.32) into Eq. (2.30), we obtain the finite difference equation associated with $x = a$,

$$\left[2 + h^2 q_0 - (2 + hp_0)h\frac{\alpha_1}{\alpha_2}\right]w_0 - 2w_1 = -h^2 r_0 - (2 + hp_0)h\frac{\alpha_3}{\alpha_2}\,. \tag{2.33}$$

2.3.2.2 Neumann boundary condition

For a Neumann boundary condition, $\alpha_1 = 0$, so the corresponding finite difference equation would becomes

$$(2 + h^2 q_o)w_0 - 2w_1 = -h^2 r_0 - (2 + hp_0)h\alpha\,, \tag{2.34}$$

where $\alpha = \alpha_3 / \alpha_2$.

Performing a similar analysis for a Robin boundary condition at $x = b$, we find the corresponding finite difference to be

$$-2w_{N-1} + \left[2 + h^2 q_N + (2 - hp_N)h\frac{\beta_1}{\beta_2}\right]w_N = -h^2 r_N + (2 - hp_N)h\frac{\beta_3}{\beta_2}\,. \tag{2.35}$$

For a Neumann boundary condition at $x = b$, the equation becomes

$$-2w_{N-1} + (2 + h^2 q_N)w_N = -h^2 r_N + (2 - hp_N)h\beta\,. \tag{2.36}$$

CHAPTER 3

WATER QUALITY MEASUREMENT USING NUMERICAL METHOD

Water pollution assessment problems arise frequently in environmental science. In this research, a finite difference method for solving a one-dimensional steady convection-diffusion equation with variable coefficients is presented.

3.1 Numerical techniques

Consider the convection - diffusion equation in the form,

$$-D_x \frac{d^2c}{dx^2} + u\frac{dc}{dx} + RC - Q = 0\,, \tag{3.1}$$

where $c(x)$ is concentration of COD at the point $x \in [a,b]\ (kg/m^3)$, $p(x)$ is flow velocity in x directions (m/s), $q(x)$ is the increasing rate of substance concentration due to a source (Kg/m^3s), and $r(x)$ the substance decay rate (s^{-1}).

First, we select an integer $N>0$ and divide the interval $[a,b]$ into $(N+1)$ equal subintervals, whose endpoints are the mesh points $x_i = a + ih$, for all $i = 1,2,\ldots,N+1$, where $h = (b-a)/(N+1)$. At the interior mesh points, x_i, for $i = 1,2,\ldots,N$, the differential equation to be approximated is

$$c''(x_i) = p(x_i)c'(x_i) + q(x_i)c(x_i) + r(x_i). \tag{3.2}$$

Expanding y in a third Taylor polynomial about x_i evaluated at x_{i+1} and x_{i-1}, we have

$$c(x_{i+1}) = c(x_i + h) = c(x_i) + hc'(x_i) + \frac{h^2}{2}c''(x_i) + \frac{h^3}{6}c'''(x_i) + \frac{h^4}{24}c^{(4)}(\xi_i^+), \tag{3.3}$$

for some ξ_i^+ in (x_i, x_{i+1}), and

$$c(x_{i-1}) = c(x_i - h) = c(x_i) - hc'(x_i) + \frac{h^2}{2}c''(x_i) - \frac{h^3}{6}c'''(x_i) + \frac{h^4}{24}c^{(4)}(\xi_i^-), \tag{3.4}$$

for some ξ_i^- in (x_{i-1}, x_i), assuming $c \in C^4[x_{i-1}, x_{i+1}]$. If these equations are added, the terms involving $c'(x_i)$ and $c'''(x_i)$ are eliminated and simple algebraic manipulation gives

$$c''(x_i) = \frac{1}{h^2}[c(x_{i+1}) - 2c(x_i) + c(x_{i-1})] - \frac{h^2}{24}\left[c^{(4)}(\xi_i^+) + c^{(4)}(\xi_i^-)\right] \tag{3.5}$$

The intermediate value theorem (Bradie, 2006) can be used to simplify this even further:

$$c''(x_i) = \frac{1}{h^2}\left[c(x_{i+1}) - 2c(x_i) + c(x_{i-1})\right] - \frac{h^2}{12}c^{(4)}(\xi_i), \tag{3.6}$$

for some ξ_i in (x_{i-1}, x_{i+1}). A centered-difference formula for $y'(x_i)$ is obtained in a similar manner resulting in

$$c'(x_i) = \frac{1}{2h}\left[c(x_{i+1}) - c(x_{i-1})\right] - \frac{h^2}{6}c'''(\eta_i), \tag{3.7}$$

for some η_i in (x_{i-1}, x_{i+1}). The use of these centered-difference formulas in Eq. (2.14) results in the equation

$$\frac{c(x_{i+1}) - 2c(x_i) + c(x_{i-1})}{h^2} = p(x_i)\left[\frac{c(x_{i+1}) - c(x_{i-1})}{2h}\right] + q(x_i)c(x_i) + r(x_i)$$
$$- \frac{h^2}{12}\left[2p(x_i)c'''(\eta_i) - c^{(4)}(\xi_i)\right]. \tag{3.8}$$

For the lower bound, $i = N$, A finite difference method with truncation error of order $O(h^2)$ results by using the equation (3.8) together with the non-Dirichlet boundary conditions

$$c(a) = c, \tag{3.9}$$

$$c'(b) = T_0. \tag{3.10}$$

That is

$$C_0 = w_0 = c_0, \tag{3.11}$$

$$C_{N+1} = w_{N+1} = 2hT_0 + w_{N-1}, \tag{3.12}$$

substituting Eq. (3.12) into Eq. (3.8), where $w_i = C(x_i)$, for all $i = 1,2,3,\ldots,N-1$. For $i = N$,

$$\frac{2hT_0 + w_{N-1} - 2w_N + w_{N-1}}{h^2} = p(x_N)\left[\frac{2hT_0 + w_{N-1} - w_{N-1}}{2h}\right] + q(x_N)w_N + r(x_N), \tag{3.13}$$

$$\frac{2hT_0 + 2w_{N-1} - 2w_N}{h^2} = p(x_N)\left[\frac{2hT_0}{2h}\right] + q(x_N)w_N + r(x_N), \tag{3.14}$$

$$\frac{2hT_0 - 2w_N + 2w_{N-1}}{h^2} = p(x_N)T_0 + q(x_N)w_N + r(x_N), \tag{3.15}$$

$$2hT_0 - 2w_N + 2w_{N-1} = h^2\left[pT_0(x_N) + q(x_N)w_N + r(x_N)\right], \tag{3.16}$$

$$(-2)w_{N-1} + 2w_N + h^2q(x_N)w_N = 2hT_0 - h^2pT_0(x_N) - h^2r(x_N), \tag{3.17}$$

$$(-2)w_{N-1} + \left(2 + h^2q(x_N)\right)w_N = -h^2r(x_N) + \left(2 - hp(x_N)\right)hT_0, \tag{3.18}$$

Using the central difference method, Eq. (2.11) and (2.12) we can obtain

$$\left(\frac{2w_i - w_{i+1} - w_{i-1}}{h^2}\right) + p(x_i)\left(\frac{w_{i+1} - w_{i-1}}{2h}\right) + q(x_i)w_i = -r(x_i), \tag{3.19}$$

$$-\left(1+\frac{h}{2}p(x_i)\right)w_{i-1}+\left(2+h^2q(x_i)\right)w_i-\left(1-\frac{h}{2}p(x_i)\right)w_{i+1}=-h^2r(x_i), \tag{3.20}$$

where $w_i=c(x_i)$, for all $i=1,2,3,\dots,N-1$. For $i=N$, substituting equation (3.10) into equation (3.8), we obtain,

$$(-2)w_{N-1}+\left(2+h^2q(x_N)\right)w_N=-h^2r(x_N)+\left(2-hp(x_N)\right)h\beta\ . \tag{3.21}$$

Then the equation system equations (3.19) can be written in the matrix form

$$AW=B, \tag{3.22}$$

where,

$$A=\begin{bmatrix} \left(2+(h^2)q(x_1)\right) & -\left(1-\left(\frac{h}{2}\right)p(x_1)\right) & & & \\ -\left(1+\left(\frac{h}{2}\right)p(x_2)\right) & \left(2+(h^2)q(x_2)\right) & -\left(1-\left(\frac{h}{2}\right)p(x_2)\right) & & \\ & \cdots & \cdots & \cdots & \\ & & \cdots & \cdots & \cdots \\ & & -\left(1+\left(\frac{h}{2}\right)p(x_{N-1})\right) & \left(2+(h^2)q(x_{N-1})\right) & -\left(1-\left(\frac{h}{2}\right)p(x_{N-1})\right) \\ & & & -2 & 2+(h^2)q(x_N) \end{bmatrix},$$

$$W=\begin{Bmatrix} w_1 \\ w_2 \\ \vdots \\ w_{N-1} \\ w_N \end{Bmatrix} \quad \text{and} \quad B=\begin{Bmatrix} -h^2r(x_N)+\left(1+\frac{h}{2}p(x_N)\right)w_0 \\ 0 \\ \vdots \\ 0 \\ -h^2r(x_N)+\left(2-hp(x_N)\right)hT_0) \end{Bmatrix}.$$

3.2 Numerical experiment : water pollution measurement

Assume that there is a plant which discharge waste water into the channel at the starting point 0.0 km. and that the COD concentrations of the waste water are 12 kg/m^3. The physical parameters are: diffusion coefficient $2\ m^2/s$, flow velocity $u=5-x\ m/s$, where $x\in[0,4]$, substance decay rate $3\ s^{-1}$ and rate of change of substance concentration due to a source $1\ Kg/m^3s$. The space increment size 100 m. is used for the numerical treatment. We can obtain variable coefficients of convection-diffusion equation (3.2) are

$$p(x) = \left(\frac{U}{D_x}\right) = \frac{5-x}{D_x} = \frac{5-x}{12},$$

$$q(x) = \left(\frac{R}{D_x}\right) = \frac{3}{12},$$

$$r(x) = \left(\frac{-Q}{D_x}\right) = \frac{-1}{12}.$$

By taking equation (3.1), we can obtain the approximate COD concentration in Table 3.1 and Fig. 3.1

Table 3.1 COD concentration assessment along a channel

Distance $(Km.)$	COD concentration (Kg/m^3)
0.0	12.0000
0.5	10.0833
1.0	8.4426
1.5	7.0791
2.0	5.9783
2.5	5.1294
3.0	4.5254
3.5	4.1640
4.0	4.0479

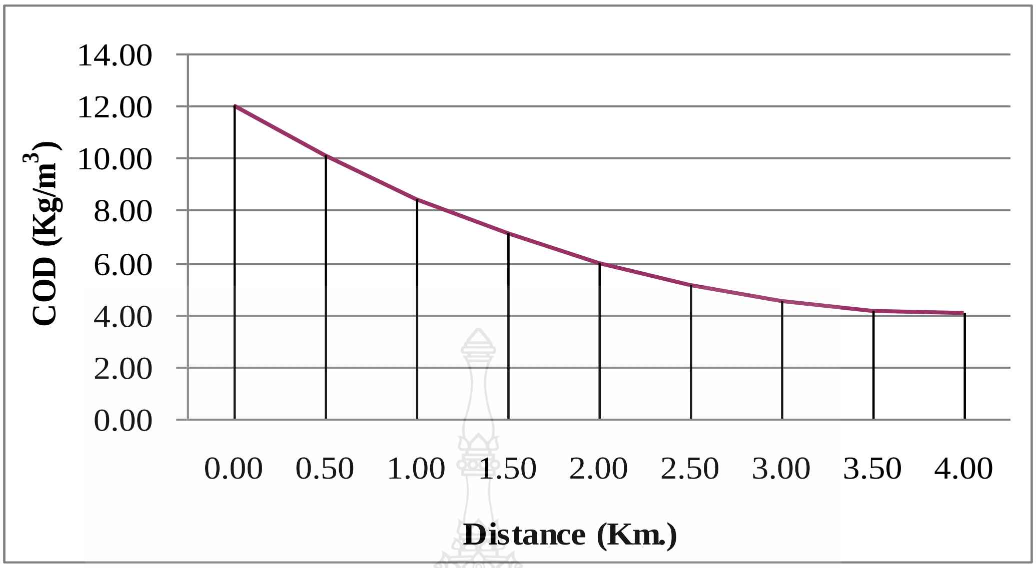

Figure 3.1 The decreasing of COD concentration along the channel

CHAPTER 4

WATER POLLUTION CONTROL USING OPTIMIZATION

Water pollution assessment problems arise frequently in environmental science. In this research, the finite difference method for solving the one-dimensional steady convection-diffusion equation with constant coefficients is presented; it is then used to optimize water treatment costs.

4.1 Optimal control of cost

Let x_β be the observation nodes and r_α be the COD concentration that is removed at inflow points. It follows that $C_\alpha - r_\alpha$ is the concentration of the pollutant after partial purification. Then

$$\tilde{C}_\beta = b_{\beta 1} g_1 + \ldots + b_{\beta\alpha}(g_\alpha - r_\alpha) + \ldots + b_{\beta N} g_N \,. \tag{4.1}$$

Let C_{ST} be the standard COD concentration. The water quality $\tilde{C}_\beta$ must be at or below the standard water quality. That is

$$\tilde{C}_\beta = \sum_{n=1}^{m} b_{\beta i} g_i + \sum_{j=1}^{n} b_{\beta\alpha_j}\left(g_{\alpha_j} - r_{\alpha_j}\right) \le C_{ST} \,, \tag{4.2}$$

where m is the number of observation points and n is the number of inflow points $(N = m + n)$. The objective function J is the cost of wastewater treatment in the system, so

$$J(x) = \sum_{j=1}^{m} w_j r_{\alpha_j} \,, \tag{4.3}$$

where w_j is the cost of waste water treatment for the required reduction of the COD concentration. The constraints are

$$\tilde{C}_\beta = \sum_{i=1}^{m} b_{\beta_i} g_i + \sum_{j=1}^{n} b_{\beta\alpha_j}\left(g_{\alpha_j} - r_{\alpha_j}\right) \le C_{ST} \,, \tag{4.4}$$

the upper bound of the control (treatment plant) is

$$r_{\alpha_j} \le u_{\alpha_j} \,, \tag{4.5}$$

the lower bound of the control (treatment plant) is

$$r_{\alpha_j} \ge l_{\alpha_j} \,, \tag{4.6}$$

the controls are non-negative, that is

$$r_{\alpha_j} \geq 0, \tag{4.7}$$

where l_{α_j} and u_{α_j} are the lower and upper bounds respectively of the points control variables. The optimal control problem is solved by the simplex method. If we assume $C = W$ on Eq. (3.22), we have the matrix form of water pollution control at sample controlled nodes number 1,3,5 as

$$C = A^{-1}B. \tag{4.8}$$

Then

$$C_1 = b_{11}(g_1 - r_1) + b_{12}g_2 + b_{13}(g_3 - r_3) + b_{14}g_4 + b_{15}(g_5 - r_5) + \ldots + b_{1N}(g_N). \tag{4.9}$$

If β is a controlled node number, the general form of control equation can be written as

$$\tilde{C}_\beta = b_{\beta 1}g_1 + \ldots + b_{\beta\alpha}(g_\alpha - r_\alpha) + \ldots + b_{\beta N}g_N. \tag{4.10}$$

In the general form, we can obtain

$$\tilde{C} = A^{-1}G, \tag{4.11}$$

where

$$\tilde{C} = \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ \vdots \\ C_N \end{Bmatrix}, \quad A^{-1} = \begin{bmatrix} b_{11} & b_{12} & \ldots & b_{1N} \\ b_{21} & b_{22} & \ldots & b_{2N} \\ b_{31} & b_{32} & \ldots & b_{3N} \\ b_{41} & b_{42} & \ldots & b_{4N} \\ b_{51} & b_{52} & \ldots & b_{5N} \\ \vdots & \vdots & & \\ b_{1N} & b_{2N} & \ldots & b_{5N} \end{bmatrix}, \quad G = B - \begin{Bmatrix} r_1 \\ 0 \\ r_3 \\ 0 \\ r_5 \\ \vdots \\ r_N \end{Bmatrix}. \tag{4.12}$$

4.2 Optimization using Microsoft Excel Solver (MacDonald,1995)

To use Excel for solve LP problems the Solver add-in must be included. Typically this feature is not installed by default when Excel is first setup on hard disk. To add this facility Tools menu we need to carry out the following steps.

1. Select the menu option Tools/ Add Ins (this will take a few moments to load the necessary file).
2. From the dialogue box presented check the box for Solver Add-In.
3. On clicking OK, we will then be able to access the Solver option from the new menu option/ Tools/ Solver (which appears below Tools/Scenarios).

To illustrate Excel Solver we will consider Hillier & Lieberman's reasonably well known example, the Wyndor Glass Co., problem (Hillier & Lieberman, 1995). The problem concerns a glass manufacturer which uses three production plants to assemble its products, mainly glass doors (x_1) and wooden frame windows (x_2). Each product requires different times in the three plants and there are certain restrictions on available production time at each plant. With this information and a knowledge of contributions to profit of the two products the management of the company wish to determine what quantities of each product they should be producing in order to maximize profits. In order words, the Wyndor Glass Co. problem is a classic, albeit very simple, product-mix problem.

The problem is formulated as the following linear program:

$$\text{Max } z = 3x_1 + 2x_2 \quad \text{(Objective)}, \tag{4.13}$$

Subject to

$$x_1 \le 4 \quad \text{(Plant One)}, \tag{4.14}$$

$$2x_2 \le 12 \quad \text{(Plant Two)}, \tag{4.15}$$

$$3x_1 + 2x_2 \le 18 \quad \text{(Plant Three)}, \tag{4.16}$$

$$x_1, x_2 \ge 0 \quad \text{(Non-negativity requirements)}, \tag{4.17}$$

where z is total profit per week,

x_1 is number of batches of doors produced per week,

x_2 is number of batches of windows produced per week.

Having formulated the problem, and may have substantially more decision variables and constraints, we can then proceed to entering it into Excel. The best approach to entering the problem into Excel is first to list in a column the names of the objective function, decision variables and constraints. We can then enter some arbitrary starting values in the cells for the decision variables, usually zero, shown in Figure 4.1. Excel will vary the values of the cells as it determines the optimal solutions. Having assigned the decision variables with some arbitrary starting values we can use these cell references explicitly in writing the formulae for the objective function and constraints, remembering to start each formula with an '='

B5 f_x =3*B9+2*B10

	A	B	C	D	E
1	**The Wyndor Glass Co. Problem**				
2					
3	**Objective**				
4					
5	Profit	0			
6					
7	**Decision variables**				
8					
9	Doors per week	0			
10	Windows per wee	0			
11					
12	**Constraints**				
13					
14	Plant One	0	4		
15	Plant Two	0	12		
16	Plant Three	0	18		
17	Non-negative 1	0	0		
18	Non-negative 2	0	0		

Figure 4.1 Setting up the problem in Excel

Entering the formula for the objective and constraints, the objective function in B5 will be given by

$$=3*B9+2*B10. \quad (4.18)$$

The constraints will be given by (putting the right hand side {RHS} value in the adjacent cells

Plant One (B14) = B9,

Plant Two (B15) = 2*B10,

Plant Three (B16) = 3*B9+2*B10,

Non-neg 1 (B17) = B9,

Non-neg 2 (B19) = B10.

On selecting the menu option Tools/ Solver the dialogue box shown in Figure Two is revealed, and if we select the objective cell before invoking Solver the correct Target Cell will be identified. This is the value Solver will attempt either to maximize or minimize.

Figure 4.2 The Solver Dialogue Box

Select whether we wish to minimize this or maximize the problem, in this case we would want to set the target cell (the objective) to a Max. Note that we can use Solver to find the outcome that will achieve a specified value for the target cell by clicking 'Value of'. In doing this we can use Solver as a glorified goal seeker. Next we enter the range of cells we want Solver to vary, the decision variables. Click on the white box and select cells B9 & B10, or alternatively type them in. Note that we can try to get Solver to guess which cells we want to vary by clicking the 'Guess' button, If we have defined our problem in a logical way Solver should usually get these right.

We can now enter the constraints by first clicking the 'Add' button. This reveals the dialogue box shown in Figure 4.3

Figure 4.3 Entering Constraints

The cell reference is to the cell containing our constraint formula, so for the Plant One constraint we enter B14. By default <= is selected but we can change this by clicking on the drop down arrow to reveal a list of other constraint types. In the right hand white box we enter the cell reference to the cell containing the RHS value, which for the Plant One constraint is cell C14. We then click ‘Add’ to add the rest of the constraints, remembering to include the non-negativity constraints.

Having added all the constraints, click ‘OK’ and the Solver dialogue box should look like that shown in Figure 4.4.

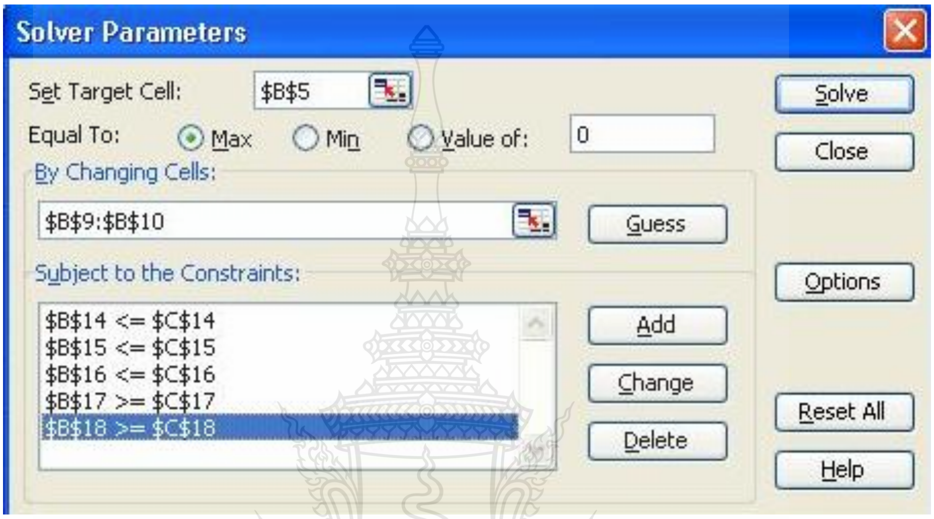

Figure 4.4 The Completed Solver Dialogue Box

Before clicking ‘Solve’ it is good practice when going LPs to go into the Options and check the ‘Assume Linear Model’ box, unless, of course, our model isn’t linear (Solver can handle most mathematical program types, including non-linear and integer problems). Doing this can speed up the length of time taken for Solver to find a solution to the problem and in fact, it will also ensure the correct result and quite importantly, provide the relevant sensitivity report. Having selected this option we are now ready to Click ‘Solve” and see Solver find the optimal values for doors and windows. On doing this, at the bottom of the screen Excel will inform we of Solver’s progress, then on finding an optimal solution the dialogue box shown in Figure Five will appear. We will also observe that Solver has altered all the values in our spreadsheet, replacing them with the optimal results.

We can use the Solver Results dialogue box to generate three reports. To select all three at once, either hold down CTRL and click each on in turn or drag the mouse over all **three.**

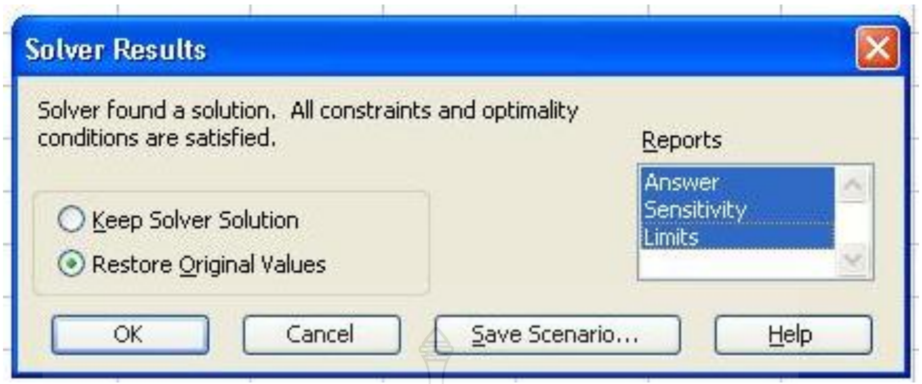

Figure 4.5 Solver Results

At the same time it's often a good idea to get Solver top restore your original values in the spreadsheet so that can return to the original problem formulation and make adjustments to the model such as altering the availability of resources. The three reports are generated in new sheets in the current workbook of Excel.

The Answer Report gives details of the solutions in this case, profit is maximized at 18 when 4 doors per week are produced and 3 windows per week- and information concerning the status of each constraint with accompanying slack/surplus values is provided. The Sensitivity Report for the Wyndor problem, which provides information about now sensitive your solution is to changes in the constraints, is shown in the figure 4.6.

	A	B	C	D	E
1	**Microsoft Excel 12.0 Sensitivity Report**				
2	**Worksheet: [งานใหม่ seminar.xlsx]Sheet1**				
3	**Report Created: 22/12/2553 21:27:12**				
4					
5					
6	Adjustable Cells				
7				**Final**	**Reduced**
8		**Cell**	**Name**	**Value**	**Gradient**
9		B9	Doors per week	4	0
10		B10	Windows per wee	3	0
11					
12	Constraints				
13				**Final**	**Lagrange**
14		**Cell**	**Name**	**Value**	**Multiplier**
15		B14	Plant One	4	0
16		B15	Plant Two	6	0
17		B16	Plant Three	18	1
18		B17	Non-negative 1	4	0
19		B18	Non-negative 2	3	0

Figure 4.6 Sensitivity Report for Wyndor

4.3 Water pollution control using optimization

Assume that there are plants A, B and C which discharge wastewater into the river at the points 0.0 km, 0.4 km and 0.8 km. and the COD concentrations of the wastewater are 1.25, 1.7418 and 1.9312 mg/l, respectively. The physical parameters are diffusion coefficient 2 m^2/s, flow velocity $u = 5 - x\ m/s$, where $x \in [0,2]$, substance decay rate 3 s^{-1} and rate of change of substance concentration due to a source 1 $mg/lday$. The legal requirement is that the plant has to decrease the COD concentration in the wastewater to less than 0.1 mg/l in the stretch from the plant A to a point 2.0 km. downstream from A. At the observation points the COD concentration must be less than 1.2 mg/l. Plants A, B and C are capable of treating the wastewater, so that the COD concentration is not greater than 1.0, 1.0 and 1.0 mg/l, respectively. The costs of wastewater treatment for the reduction by 1 mg/l of COD concentration are 200,000, 300,000 and 360,000 Baht for plants A, B and C, respectively. It turns out that the least cost of wastewater treatment is 406,339.58 Baht.

Table 4.1 Compare COD concentration at the discharge point at node 1, 3, 5 (mg/l)

Node	Unpurified Inflow	Observations	Unpurified Inflow	Observations	Purified Inflow	Observations
1	1.2500	1.2500	1.0000	1.0000	1.1500	1.1500
2	-	0.7585	-	0.6068	-	0.6978
3	1.7418	1.7418	1.0000	1.0000	1.1838	1.1838
4	-	1.7600	-	0.9709	-	1.2000
5	1.9312	1.9312	1.0000	1.0000	1.3231	1.3231
6	-	1.7516	-	0.9070	-	1.2000
7	-	1.5923	-	0.8245	-	1.0909
8	-	1.4542	-	0.7530	-	0.9963
9	-	1.3405	-	0.6941	-	0.9184
10	-	1.2581	-	0.6515	-	0.8619
11	-	1.2214	-	0.6325	-	0.8368

From the table shown that, the COD concentration at discharge points 0.0, 0.4 and 0.8 km. are 1.25, 1.7418 and 1.9312 mg/l, respectively. We can see the COD concentration at the observation points as the table from node 1 to 11. If we change the COD at discharge points from 1.25 to 1.0000, 1.7418 to 1.0000 and 1.9312 to 1.0000, it will give the COD at the observations as 1.0000, 0.6068, 1.0000, 0.9709, 1.0000, 0.9070, 0.8245, 0.7530, 0.6941, 0.6515 and 0.6325 respectively.

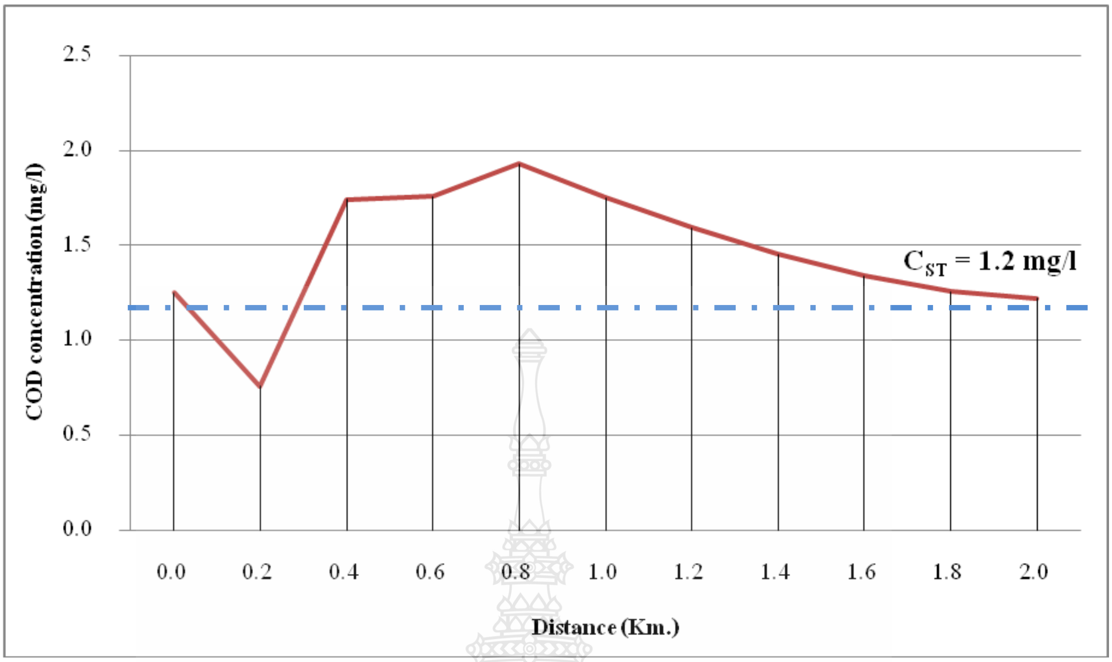

Figure 4.7 Unpurified Inflow

We also show the graph below of Unpurified Inflow. The graph shown that the COD concentration before control at the observations points are greater than the standard 1.2 mg/l.

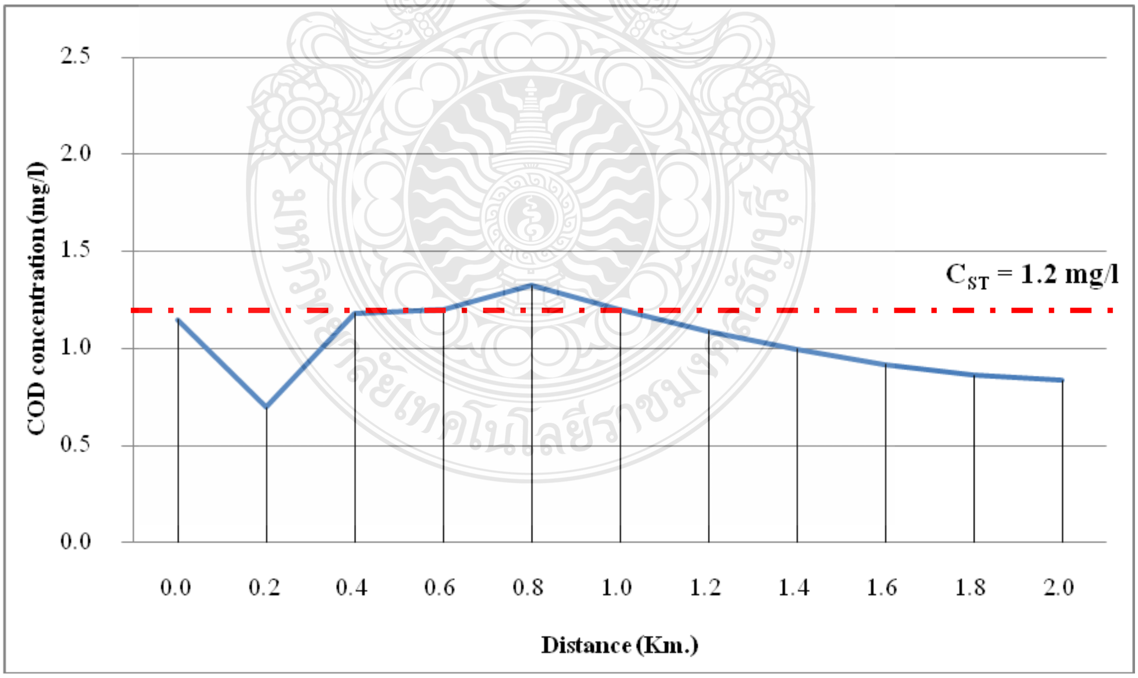

Figure 4.8 Purified Inflow

We can see that the COD concentration after control almost less than the standard 1.2 mg/l.

Table 4.2 Optimal cost of wastewater treatment

Plant	Unpurified Inflow Of COD Concentration (mg/l)	Optimal Cost of Unpurified Inflow (Baht)	Optimal Reduction Of COD Concentration (mg/l)	Optimal Cost of Reduction (Baht)
A	1.0000	200,000.00	0.1000	20,000.00
B	1.0000	300,000.00	0.5580	167,412.41
C	1.0000	360,000.00	0.6081	218,927.17
	Cost (Baht)	860,000.00	Minimum cost (Baht)	406,339.58

From the table, the plant A can reduce COD concentration 0.1000 mg/l at the cost 20,000 baht, plant B can reduce COD concentration 0.5580 mg/l at the cost 167,412.41 baht and plant C can reduce COD concentration 0.6081 mg/l at the cost 218,927.17 baht. So the minimum cost of treatment in the system is 406,339.58 baht.

The COD concentration at discharge point at plant A, B and C are 1.0000, 1.0000 and 1.0000 mg/l. The cost are 200,000, 300,000 and 360,000 baht respectively. The total cost in the system is 860,000 baht that is much more the cost of we control 406,339.58 baht.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

We have established a simulation process by finite difference method. By the numerical solutions, it can be obtained that the COD concentration along a channel. These mean that the trend of concentration along the channel is lower than the discharge concentration. We can conclude that the water pollution levels can be reduced to an agreed standard at least cost.

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