



## A new hybrid projection method for variational inclusion problems and generalized mixed equilibrium problems

W. Kumam<sup>a,b</sup>, C. Jaiboon<sup>c</sup>, P. Kumam<sup>b,d,\*</sup>, A. Singta<sup>a,†</sup>

<sup>a</sup> Department of Mathematics and Computer Science Faculty of Science and Technology, Rajamangala University of Technology Thanyaburi (RMUTT), Pathumthani, Thailand

<sup>b</sup> Centre of Excellence in Mathematics, CHE, Si Ayutthaya Rd., Bangkok, Thailand

<sup>c</sup> Department of Mathematics, Faculty of Liberal Arts, Rajamangala University of Technology Rattanakosin (RMUTR), Bangkok, Thailand

<sup>d</sup> Department of Mathematics, King Mongkut's University of Technology Thonburi (KMUTT), Bangkok, Thailand

\*Corresponding author, e-mail : poom.kum@kmutt.ac.th

†Presenting author

**ABSTRACT:** The purpose of this paper is to consider a shrinking projection method for finding a common element of the set of solutions of generalized mixed equilibrium problems, the set of fixed points of a finite family of quasi-nonexpansive mappings and the set of solutions of variational inclusion problems. Then, we prove a strong convergence theorem of the iterative sequence generated by the shrinking projection method under some suitable conditions in a real Hilbert space. Our results improve and extend recent results announced by Peng et al. (2008), Takahashi et al. (2008), Takahashi and Takahashi (2008) and many others.

**KEYWORDS:** Generalized mixed equilibrium problem, Fixed point, Variational inequality, Nonexpansive mapping, Inverse-strongly monotone mapping.

### References

- [1] G.L. Acedoa and H.K. Xu, Iterative methods for strict pseudo-contractions in Hilbert spaces, *Nonlinear Analysis*, **67** (2007), 2258–2271.

This project was supported by the National Research Council of Thailand to RMUTT/ 2009–2010.

- [2] S. Atsushiba and W. Takahashi, Strong convergence theorems for a finite family of nonexpansive mappings and applications, *Indian Journal Mathematics*, **41** (1999), 435–453.
- [3] E. Blum and W. Oettli, From optimization and variational inequalities to equilibrium problems, *The Mathematics Student*, **63** (1994), 123–145.
- [4] H. Brézis, Opérateur maximaux monotones, in *Mathematics Studies*, vol. 5, North-Holland, Amsterdam, The Netherlands, 1973.
- [5] L. C. Ceng, D. R. Sahu and J. C. Yao, Implicit Iterative Algorithms for Asymptotically Nonexpansive Mappings Nonexpansive Mappings in the Intermediate Sense and Lipschitz-Continuous Monotone Mappings, *Journal of Computational and Applied Mathematics*, **233** (2010), 2902–2915.
- [6] L. C. Ceng and J. C. Yao, A Relaxed Extragradient-like Method for a Generalized Mixed Equilibrium Problem, a General System of Generalized Equilibria and a Fixed Point Problem, *Nonlinear Analysis Series A: Theory, Methods & Applications*, **72** (2010), 1922–1937.
- [7] L. C. Ceng, A. Petruşel and J. C. Yao, Iterative Approaches to Solving Equilibrium Problems and Fixed Point Problems of Infinitely Many Nonexpansive Mappings, *Journal of Optimization Theory and Applications*, **143** (2009), 37–58.
- [8] F. Cianciaruso, G. Marino, L. Muglia, and Y. Yao, A Hybrid Projection Algorithm for Finding Solutions of Mixed Equilibrium Problem and Variational Inequality Problem, *Fixed Point Theory and Applications*, vol. 2010, Article ID 383740, 19 pages.
- [9] O. Chadli, Z. H. Liu and J. C. Yao, Applications of equilibrium Problems to a Class of Noncoercive Variational nequalities, *Journal of Optimization Theory and Applications*, **132** (2007), 89–110.
- [10] O. Chadli, S. Schaible, and J.C. Yao, Regularized equilibrium problems with application to noncoercive hemivariational inequalities, *Journal of Optimization Theory and Applications*, vol. 121, no. 3, pp. 571–596, 2004.
- [11] O. Chadli, N. C. Wong, and J.C. Yao, Equilibrium problems with applications to eigenvalue problems, *Journal of Optimization Theory and Applications*, **117**(2) (2003), 245–266.
- [12] P. Cholamjiak and S. Suantai, A New Hybrid Algorithm for Variational Inclusions, Generalized Equilibrium Problems, and a Finite Family of Quasi-Nonexpansive Mappings, *Fixed Point Theory and Applications*, vol. 2009, Article ID 350979, 20 pages.
- [13] C. Jaiboon and P. Kumam, A general iterative method for addressing mixed equilibrium problems and optimization problems, *Nonlinear Analysis Series A: Theory, Methods & Applications*, **73** (2010), 1180–1202.

- [14] C. Jaiboon and P. Kumam, Strong Convergence for Generalized Equilibrium Problems, Fixed Point Problems and Relaxed Cocoercive Variational Inequalities, *Journal of Inequalities and Applications*, vol. 2010, Article ID 728028, 43 pages.
- [15] C. Jaiboon, W. Chantarangsi and P. Kumam, A convergence theorem based on a hybrid relaxed extragradient method for generalized equilibrium problems and fixed point problems of a finite family of nonexpansive mappings, *Nonlinear Analysis: Hybrid Systems*, **4** (2010), 199–215.
- [16] P. Katchang and P. Kumam, A general iterative method of fixed points for mixed equilibrium problems and variational inclusion problems, *Journal of Inequalities and Applications*, vol. 2010, Article ID 370197, 25 pages.
- [17] P. Kumam and C. Jaiboon, A new hybrid iterative method for mixed equilibrium problems and variational inequality problem for relaxed cocoercive mappings with application to optimization problems, *Nonlinear Analysis: Hybrid Systems*, **3** (2009), 510–530.
- [18] I. V. Konnov, S. Schaible, and J.C. Yao, Combined relaxation method for mixed equilibrium problems, *Journal of Optimization Theory and Applications*, **126**(2) (2005), 309–322.