Effects of Variable Viscosity and Thermal Conductivity of Unsteady Mixed Convection Flow at the Stagnation Point and an Applied Magnetic Field

Dr. Jatindra Lahkar
Department of Mathematics, Digboi College, Digboi – 786171, Assam, India.
e-mail *: j_lahkar@yahoo.co.in.*

Abstract: In this paper the effects of variable viscosity and variable thermal conductivity of unsteady laminar incompressible mixed convection flow of an electrically conducting fluid at the stagnation of a two-dimensional body and an axi-symmetric body in the presence of applied magnetic field is investigated. Both prescribed wall temperature and prescribed heat flux condition have been considered. The problem was studied under the effects of variable viscosity, and variable thermal conductivity. Using a similarity transformation, the governing fundamental equations are approximated by a system of nonlinear ordinary differential equations. The resultant system of ordinary differential equations is then solved numerically using Runge-Kutta shooting method with guessed initial conditions. Details of the velocity and temperature fields as well as the local skin friction and the local Nusselt number for various values of the parameters of the problem are presented. The results presented, demonstrate quite clearly that Ge and Gr, which are indicator of the variation of viscosity and thermal conductivity with temperature have a substantial effect on the drag and heat transfer characteristics as well as the velocity and temperature distributions within the boundary layer.

Mathematics Subject Classification: 76.

Key words: variable viscosity, variable thermal conductivity, viscosity parameter, conductivity parameter, mixed convection.

1. Introduction:

In the study of convective heat transfer, it is customary to treat the problem as either pure forced convection or pure free convection. The study of mixed convection flow arises in many transport processes in nature and in engineering devices such as nuclear reactors cooled during emergency shutdown, solar central receivers exposed to winds, electronic devices cooled by fans, heat exchangers placed in a low-velocity environment. The mixed convection in stagnation flows become important when the buoyancy forces due to the temperature difference between the wall and free stream become large. In such a situation the flow and thermal fields are not symmetric with respect to the stagnation line.

The mixed convection around heated vertical surfaces has been studied by several authors. Sparrow et al [1] studied the combined and free convection flow and heat transfer about a non-isothermal body subject to a non-uniform free stream velocity. A similarity solution for mixed convection from horizontal impermeable surface in saturated porous media was studied by Cheng [2]. Mixed convection induced in a gas flowing past a hot horizontal flat plate was studied by Pop et al [3]. Studies on forced and free mixed convection boundary layer flow with uniform suction or injection on a vertical plate was carried out by Watanabe [4]. Laminar mixed convection over horizontal flat plate with power-law variation in surface temperature was studied by Risbeck et al [5].

Also similarity solution of mixed convection around vertical surfaces has been studied by Ramachandran et al [6], Mahmood and Merkin [7] and Merkin and Mahmood [8]. Non-similarity solutions for mixed convection from horizontal surface in a porous medium variable surface heat flux were investigated by Aldoss et al [9]. All of the above studies deal with steady flow. The analogous unsteady case was studies by Surma Devi et al [10]. Also unsteady mixed convection flow at the stagnation point was studied by Kumari et al [11], Ibrahim [12] Ahmad and Nazar [13], Ali F. M. et al [14] in various ways.

All the above discussions are based on the constant physical properties. Lawal and Majumder [15] and Etemad et al [16] showed that assumption of constant viscosity exhibits substantial deviation from variable viscosity results. Also the viscous dissipation can be very significant. The effects of temperature dependent viscosity and viscous dissipation on laminar convection heat transfer of a semi-circular duct were examined and discussed by Etemad et al [17]. Kafoussias et al [18] have studied the thermal-diffusion and diffusion-thermo effects on the mixed free-forced convective and mass transfer steady laminar boundary-layer flow along a vertical semi-infinite isothermal flat plate, when the viscosity of the fluid varies with temperature Sharma and Sing [19] investigated the effects of variable thermal conductivity and heat source / sink on MHD flow near a stagnation point on a linearly stretching sheet.

The aim of this work is to study the effects of variable viscosity and variable thermal conductivity of unsteady laminar incompressible mixed convection flow of an electrically conducting fluid at the stagnation of a two-dimensional body and an axi-symmetric body in the presence of applied magnetic field. The effect of induced magnetic field has been included in the analysis. Both prescribed wall temperature and prescribed heat flux condition have been considered. It has been found that if the free stream velocity, applied magnetic field and square root of wall temperature vary inversely as a function of time (i.e. $(1-\lambda t^*)^{-1}$), viscosity and thermal conductivity of the fluid vary inversely as a linear function of temperature, the governing boundary layer equations admit a local similarity solutions. The resulting ordinary differential equations have been solved by using Runge-Kutta shooting method with guessed initial conditions. Particular cases of the present results have been compared with those of [11].

2. Mathematical Formulation of the Problem:

Consider the unsteady laminar incompressible viscous electrically conducting fluid flow at the stagnation point of a two-dimensional and axi-symmetric body. A magnetic field H_0 is imposed parallel to the surface (i.e. along the x-axis) outside the boundary layer. We assumed that both viscous and magnetic Reynolds numbers are sufficiently large for momentum and magnetic boundary layers have to be developed. The effects of the induced magnetic field, viscous dissipation and Joule heating have been included in the analysis. However, the Hall effect is neglected. It is assumed that there is no applied voltage, which implies the absence of the electric field (i.e. E = 0). The electrical currents flowing in the fluid give rise to an induced magnetic field which would exist if the fluid were an electrically insulator. Here it is assumed that the normal component of the induced magnetic field H_2 vanishes at the wall and the parallel component H_1 is approaches its given value H_0 [20]. The unsteadiness of the flow, temperature and magnetic fields is caused by the time dependent free stream velocity, applied magnetic field and wall temperature (heat flux). The surface is assumed to have either prescribed temperature or it is subjected to prescribed heat flux distribution. The x-axis is considered along the surface and y-axis is perpendicular to the plate, u and v are the viscosity components along the increasing direction of x and y-axis respectively. We assumed Temperature T_w is wall temperature and T_∞ is the temperature of the fluid in the free stream which is assumed to be constant. The fluid property variations with temperature are limited to (i) viscosity (ii) thermal conductivity and (iii) density variation. The influence of variation of density with temperature is restricted to the body force term only with accordance with the Boussinesq's approximation. Under the forgoing assumptions, the boundary layer equations governing the unsteady mixed convection flow under Boussinesq's approximations in the neighbourhood of the stagnation point of a two dimensional and an axi-symmetric body can be expressed as [11].

$$\frac{\partial}{\partial x}(x^k u) + \frac{\partial}{\partial y}(x^k v) = 0 \tag{1}$$

$$\frac{\partial}{\partial x}(x^k H_1) + \frac{\partial}{\partial y}(x^k H_2) = 0 \tag{2}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\rho_\infty{}^{-1}\frac{\partial p}{\partial x} + \rho_\infty{}^{-1}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + (\mu_0 \rho_\infty{}^{-1})\left[H_1\frac{\partial H_1}{\partial x} + H_2\frac{\partial H_2}{\partial y}\right]$$
$$+ g\bar{\beta}(T - T_\infty) \tag{3}$$

$$\frac{\partial H_1}{\partial t} + u\frac{\partial H_1}{\partial x} + v\frac{\partial H_1}{\partial y} - H_1\frac{\partial u}{\partial x} - H_2\frac{\partial u}{\partial y} = \alpha_1\frac{\partial^2 H_1}{\partial y^2} \tag{4}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = (\rho_\infty c_p)^{-1}\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \sigma^{-1}\left(\frac{\partial H_1}{\partial x}\right)^2 \tag{5}$$

The initial and boundary conditions are given by

$$\left.\begin{array}{l} u(x, y, 0)=u_i(x, y),\ v(x, y, 0)=v_i(x, y),\ H(x, y, 0)=H_i(x, y), \\ H_2(x, y, 0)=H_{2i}(x, y),\ T(x, y, 0)=T_i(x, y), \end{array}\right\} \tag{6}$$

$$\left.\begin{array}{l} u(x, 0, t)=0,\ v(x, 0, t)=0,\ \dfrac{\partial}{\partial y}[H_1(x,0,t)] = 0,\ H_2(x, 0, t)=0,\ T(x, 0, t)=T_w, \\ u(x, \infty, t)= u_e,\ H_1(x, \infty, t)=H_0,\ T(x, \infty, t)=T_\infty . \end{array}\right\} \tag{7}$$

In the above equations ρ_∞ is the free stream density, μ the viscosity, g the gravitational acceleration, T is the temperature inside the boundary layer, $\bar{\beta}$ the volumetric coefficient of thermal expansion, p the static pressure, c_p the specific heat at constant pressure, K the thermal conductivity, α_1 the dimensional parameter associated with the magnetic Prandtl number, Pr the Prandtl number, σ the electrical conductivity, μ_0 the magnetic permeability, u_e the velocity the velocity at the edge of the boundary layer.

In order to reduce the set of partial differential equations (1 - 5) to a set of ordinary differential equations, let us apply the following transformations

$$\left.\begin{array}{l} \eta=(1+k)^{1/2}(a/\nu_\infty)^{1/2}(1-\lambda t^*)^{-1/2}y,\ t^* = at,\ \lambda t^* < 1,\ x^k u = \dfrac{\partial \psi}{\partial y},\ -x^k v = \dfrac{\partial \psi}{\partial x}, \\ -x^k H_1 = \dfrac{\partial \phi}{\partial y},\ -x^k H_2 = \dfrac{\partial \phi}{\partial x},\ u_e= ax\,(1-\lambda t^*)^{-1},\ H_0= bx\,(1-\lambda t^*)^{-1}, \\ u_e= u_s\,(1-\lambda t^*)^{-1},\ u_s = ax,\ a = (\dfrac{\partial u_e}{\partial x})_{t^8=0} = \dfrac{du_s}{dx}, b = (\dfrac{\partial H_0}{\partial x})_{t^8=0}, \end{array}\right\} \tag{8a}$$

$$\psi=(a\nu_\infty)^{1/2}x^{(1+k)}(1+k)^{-1/2}(1-\lambda t^*)^{-1/2}f,\ \phi=b(\nu_\infty/a)^{1/2}x^{(1+k)}(1+k)^{-1/2}(1-\lambda t^*)^{-1/2}s, \tag{8b}$$

$$\left.\begin{array}{l} u=ax(1-\lambda t^*)^{-1}f',\ -v=b(a\nu_\infty)^{1/2}(1+k)^{-1/2}(1-\lambda t^*)^{-1/2}f, \\ H_1=bx(1-\lambda t^*)^{-1}s',\ -H_2=b(a\nu_\infty)^{1/2}(1+k)^{-1/2}(1-\lambda t^*)^{-1/2}s, \end{array}\right\} \tag{8c}$$

$$\left.\begin{array}{l} \mu=\mu_\infty/[1+\gamma(T - T_\infty)],\ K = K_\infty/[1+\kappa(T - T_\infty)],\ 1/\mu = A(T - Te), \\ 1/K = B(T - Tr),\ A = \gamma/\mu_\infty,\ Te = T_\infty - 1/\gamma, B = \kappa/K_\infty,\ Tr = T_\infty - 1/\kappa. \end{array}\right\} \tag{8d}$$

For prescribed wall temperature, the dimensionless temperature G is defined as

$$T - T_\infty = (T_w - T_\infty)G,\ T_w - T_\infty = (T_{w0} - T_\infty)(1 - \lambda t^*)^{-2}. \tag{8e}$$

For prescribed heat flux, G is defined as

$$\left.\begin{array}{l} T - T_\infty = G(q_w/K)(1+k)^{-1/2}(a/\nu_\infty)^{-1/2}(1 - \lambda t^*)^{-1/2}, \\ q_w = q_{w0}(1+k)^{-1/2}(1 - \lambda t^*)^{-5/2},\ q_w = -(K\dfrac{\partial T}{\partial y})_w \end{array}\right\} \tag{8f}$$

where, t^* the dimensionless time, a, b the gradients of free stream velocity and applied magnetic field respectively when $t^* = 0$, λ the dimensionless parameter measures the unsteadiness in the free stream velocity, φ the dimensional function, ψ the dimensional stream function, u_s the velocity at steady state, f the dimensionless stream function, f' the dimensionless velocity, s the dimensionless induced magnetic field in *y*-direction, s' the dimensionless induced magnetic field in *x*-direction, G the dimensionless temperature. A, Te, Tr are constants and their values depend on the reference state and the thermal properties of the fluid, i.e. γ and κ, α the reciprocal of magnetic Prandtl number.

Using above transformations, we find that equations (1) and (2) are satisfied identically and equations (3 – 5) reduce to

$$f''' = \frac{f''G'}{G-Ge} + \frac{G-Ge}{Ge}[ff'' + (1+k)^{-1}(1-f'^2) + (1+k)\beta(s'^2-1) - \beta ss'' + (1+k)^{-1}\lambda_1 G + (1+k)^{-1}\lambda(1-f'-.5\eta f'')] \quad (9)$$

$$s''' = \frac{1}{\alpha}[\lambda(1+k)^{-1}(s'+.5\eta s'') - (fs'' - f''s)] \quad (10)$$

$$G'' = \frac{G'^2}{G-Gr} + \frac{G-Gr}{Gr}\Pr[fG' + Ec(\alpha\beta s''^2 - \frac{Ge}{G-Ge}f''^2) - \lambda(1+k)^{-1}(2G+.5\eta G')] \quad (11)$$

The boundary conditions become:

$$\left.\begin{aligned} &\text{at } \eta = 0;\ f(0) = f'(0) = s(0) = s'(0),\ G(0) = 1 \text{ for (PWT)},\\ &\qquad\qquad G'(0) = -1 \text{ for (PHF)},\\ &\text{at } \eta \to \infty;\ f'(\infty) = 1,\ s'(\infty) = 1,\ G(\infty) = 0. \end{aligned}\right\} \quad (12)$$

where, prime denote partial differentiation with respect to the variable η and the dimensionless parameters are defined as

$Pr = \rho_\infty \nu_\infty c_p / K_\infty$ (Prandtl number), $Re_x = u_s x / \nu_\infty$ (Local Reynolds number)

$\beta = (\mu_0 b_2 / a_2)$ (Magnetic number).

In case of PWT,

$Ec = u_s^2 / [c_p(T_{w0} - T_\infty)]$ (Eckert number) and

$Gr_x = g\bar{\beta}(T_{w0} - T_\infty)x^3 / \nu_\infty^{\ 2}$ (Local Grashof number).

In case of PHF,

$Ec = u_s^2 / [c_p(q_{w0}/K)(\nu_\infty/a)^{1/2}]$ and $Gr_x = g\bar{\beta}q_{w0}(\nu_\infty/a)^{1/2}x^3/[\nu_\infty^{\ 2}(1+k)]$

$\lambda_1 = Gr_x / Re_x^2$ (Temperature buoyancy parameter).

$Ge = \dfrac{T_e - T_\infty}{T_w - T_\infty} = -\dfrac{1}{\gamma(T_w - T_\infty)}$ (Viscosity Parameter),

$Gr = \dfrac{T_r - T_\infty}{T_w - T_\infty} = -\dfrac{1}{\kappa(T_w - T_\infty)}$ (Thermal conductivity parameter).

Equations (9 – 11), with boundary conditions (12) described the unsteady mixed convective flow of an electrically conducting fluid at the stagnation of a two dimensional body (k = 0) and an axi-symmetric body (k = 1) in the presence of an applied magnetic field, with temperature dependent viscosity and thermal conductivity.

The skin friction and heat transfer co-efficient can be expressed in the form

$$C_f = \frac{2\tau_w}{\rho_\infty u_e^{\ 2}} = (1+k)^{1/2}(\overline{\mathrm{Re}}_x)^{-1/2}\frac{2Ge}{Ge-1}f_w''(0, Ge, Gr), \quad (13)$$

$$Nu = -x(\frac{\partial T}{\partial y})_w/(T_w - T_\infty) = -(1+k)^{1/2}(\overline{\mathrm{Re}}_x)^{1/2}G_w'(0, Ge, Gr), \quad \text{for PWT.} \quad (14)$$

$$\overline{\mathrm{Re}}_x = \frac{u_e x}{\nu_\infty}.$$

For prescribed heat flux, the heat transfer co-efficient Nu is given by

$$Nu = -q_w x / K(T_w - T_\infty) = (1+k)^{1/2} (\overline{\mathrm{Re}}_x)^{1/2} G_w^{-1}(0, Ge, Gr) \text{ where } q_w = -(K \frac{\partial T}{\partial y})_w . \quad (15)$$

3. Numerical Results and Discussion:

The problem under consideration is obtained by the system of equations (9 –11) and the boundary conditions (12) and constitutes a boundary value problem of ordinary differential equations solved by numerically using the Runge-Kutta Shooting method for different values of the parameters. To verify the proper treatment of the problem the situation have been compared with those of the corresponding constant viscosity and thermal conductivity cases by setting $\gamma = 0$ and $\kappa=0$. Thus Kumeri et al [11] have obtained $f''_w(0) = 3.670528$ and $G_w(0) = 2.555908$ when k = 1, and Ec = 0.05 for prescribed heat flux and $f''_w(0) = 1.880254$ and $G'_w(0) = 0.342159$ for prescribed wall temperature. Our results for $f''_w(0)$ and $G_w(0)$ for $\gamma = 0$, $\kappa=0$ are 3.663157 and 2.540160 respectively for prescribed heat flux and 1.883360 and 0.342996 respectively for prescribed wall temperature. Therefore our results are in very good agreement with those of [11]. We have compared our results for skin friction factor $f''_w(0)$, the value of $s''_w(0)$ = and heat transfer factor $G_w(0)$ for prescribed heat flux and $G'_w(0)$ for prescribed wall temperature for constant properties with those of [11]. The results are found to be in excellent agreement. The comparison is given in Table-1 and Table-2 respectively.

Table - 1 Comparison of skin friction f''_w, heat transfer G_w and missing values of s''_w for linear heat flux and for β=0.5, λ=-0.5, λ_1=5, α=10, Pr=0.7 and Ge=Gr=5000.

		Present Calculation			*M. Kumeri[11]*		
K	Ec	f''_w	s''_w	G_w	f''_w	s''_w	G_w
1	0	3.403258	0.320223	2.356546	3.413083	0.320081	2.356948
1	0.01	3.452033	0.321817	2.390418	3.460554	0.321640	2.391053
1	0.05	3.663157	0.328432	2.540160	3.670528	0.327402	2.555408
1	0.1	3.970824	0.322410	2.782407	3.962166	0.322875	2.777799
0	0	6.342400	0.497871	2.416501	6.343077	0.497850	2.415956
0	0.01	6.527592	0.501475	2.509657	6.528326	0.501453	2.509075
0	0.05	7.382945	0.517064	2.954760	7.383960	0.517038	2.553960
0	0.1	8.807564	0.540010	3.744694	8.809138	0.539973	3.743490

Table -2. Comparison of skin friction f''_w, heat transfer G'_w and missing values of s''_w for prescribe wall temperature and for β=0.5, λ=-0.5, λ_1=5, α=10, Pr=0.7 and Ge=Gr=5000.

		Present Calculation			*M. Kumeri[11]*		
K	Ec	f''_w	s''_w	G'_w	f''_w	S''_w	G'_w
1	0	1.502985	0.252749	-0.212176	1.512356	0.255911	-0.225760
1	0.01	1.575239	0.259934	-0.131928	1.572358	0.258752	-0.131729
1	0.05	1.863360	0.282101	0.342996	1.880254	0.281230	0.342159
1	0.1	2.307485	0.317017	1.208611	2.315472	0.316226	1.191167
0	0	3.523751	0.378996	-0.209259	3.524127	0.378997	-0.209292
0	0.01	3.672190	0.384960	0.072703	3.672576	0.384961	0.072705
0	0.05	4.586000	0.492248	2.012264	4.586422	0.492256	2.012499
0	0.1	5.911982	0.538911	5.912125	5.912353	0.538911	5.912680

Tables 3 – 6 contain the numerical results for the friction factor, missing values of $s''_w(0)$ and Nusselt number for Pr = .7, Ec =.1, λ = -.5, λ_1 = 5, k = 0 (for two dimensional flow), k=1 (for axi-symmetric flow) and for various values of Ge and Gr for the case of prescribed wall temperature and for prescribed heat flux. From Table-3, it is observed that when k=0, $f''_w(0)$ decreases and when k=1, $f''_w(0)$ increases for increasing values of the thermal conductivity parameter Gr. Also, for both k=0, k=1, $s''_w(0)$ decreases as Gr increases. Where as for increasing values of Gr, $G'_w(0)$ increases. It is observed from the Table- 4, that $f''_w(0)$ increases for both k=0, 1, for increasing values of the viscosity parameter Ge and $G'_w(0)$ decreased for both k=0, 1. But for two-dimensional flows (k=0) the values of $s''(0)$ increases where as for axi-symmetric flows (k=1) the values of $s''_w(0)$ decreases for in creasing values of Ge.

Table -3. Numerical results for the friction factor f''_w, missing values of s''_w, Nusselt number G'_w for Pr=0.7, Ec=0.1, λ=-0.5, λ_1=5, β=1, Ge= -5, α=10, k=0,1 and various values of Gr. (For prescribe wall temperature)

	k=0			k=1		
Gr	f''_w	s''_w	G'_w	f''_w	s''_w	G'_w
-10	3.830364	0.397503	0.171150	1.032684	0.195807	0.312630
-4	3.810523	0.395444	0.203884	0.078997	0.194314	0.386285
-1	3.740696	0.386582	0.413981	1.356235	0.190208	0.802948
-0.8	3.726379	0.384460	0.487638	1.451723	0.189619	0.951976
-0.4	3.689132	0.376950	0.871990	1.844415	0.189070	1.674214
-0.2	3.592809	0.368939	1.687395	2.454312	0.189002	3.105249
2	3.955188	0.389634	0.021230	2.678563	0.212356	0.165254
4	3.900473	0.405594	0.079489	2.752567	0.3156782	0.213564
10	3.867368	0.401927	0.114751	2.952653	0.354561	0.265785

Table -4. Numerical results for the friction factor f''_w, missing values of s''_w, Nusselt number G'_w for Pr=0.7, Ec=0.1, λ=-0.5, λ_1=5, β=1, Gr= -5, α=10, k=0,1 and various values of Ge. (For prescribe wall temperature).

	k=0			k=1		
Ge	f''_w	s''_w	G'_w	f''_w	s''_w	G'_w
-10	3.621745	0.383624	0.200748	0.910345	0.207921	0.283216
-6	3.744232	0.384625	0.189282	0.944157	0.207845	0.281244
-2	4.316871	0.388413	0.142475	1.107791	0.207811	.0274652
-1	5.080724	0.392245	0.092836	1.338052	0.207615	0.270245
-0.8	5.430846	0.393628	0.073691	1.446874	0207505	0.269147
-0.4	6.966279	0.398145	0.006846	1.939248	0.207122	0.267362
2	2.341041	0.367684	0.380834	0.295671	0.204394	0.425673
4	3.097632	0.378632	0.257941	0.865412	.204286	0.267512
10	3.234315	0.350176	0.241573	0.925146	0.204215	0.254122

From Table - 5, it is observed that for k=0 and k=1, $f''_w(0)$ increases with increasing values of Ge and $s_w''(0)$ and $G_w(0)$ decreases for increasing values of Ge. Table- 6 shows that for k=0 and k=1, $f''_w(0)$, $s_w''(0)$ and $G_w(0)$ decreases as Gr increases.

Table - 5. Numerical results for the friction factor f''_w, missing values of s''_w, Nusselt number G_w for Pr=0.7, Ec=0.1, λ=-0.5, λ_1=5, β=1, Gr= -5, α=10, k=0,1 and various values of Ge. (For prescribe heat flux).

	k=0			k=1		
Ge	f''_w	s''_w	G_w	f''_w	s''_w	G_w

-10	9.619937	0.533384	3.547195	4.184410	0.249054	3.154327
-6	10.178452	0.533184	3.438076	4.538663	0.247607	3.104961
-2	11.919580	0.531962	2.907609	6.075367	0.241773	2.976681
-1	14.063255	0.430618	2.642103	7.969466	0.236921	2.894881
-0.8	15.286194	0.412989	2.600216	8.285649	0.224568	2.715476
-0.4	19.025487	0.385478	2.541237	10.548712	0.194578	2.354127

Table -6. Numerical results for the friction factor f''_w, missing values of s''_w, Nusselt number G_w for Pr=0.7, Ec=0.1, λ=-0.5, λ_1=5, β=1, Ge= -5, α=10, k=0,1 and various values of Gr. (For prescribe heat flux).

	k=0			k=1		
Gr	f''_w	s''_w	G_w	f''_w	s''_w	G_w
-10	10.672382	0.541733	3.342084	4.459164	0.294910	2.882217
-6	10.493620	0.535549	3.323980	4.340650	0.287548	2.837981
-2	9.916869	0.420526	3.311375	4.009927	0.273423	2.845127
-1	10.259451	0.410250	3.617110	4.078463	0.260021	3.036928
-0.8	10.590988	0.407112	3.823042	3.638283	0.306451	2.813501
-0.4	14.081979	0.390316	5.555467	4.082100	0.297064	3.346185

For prescribed wall temperature case ($T_w>T_0$) the velocity and temperature profiles (f',G) showing the effects of viscosity parameter *Ge* and the thermal conductivity parameter *Gr* are given in Figures 1 – 4. From Figures 1- 2, it is observed that the velocity increases with the dimensionless distance η from the flat surface, takes its maximum values inside the boundary layer before decreasing asymptotically to its free stream value. This classical mixed convection profile is more evident for higher values of the viscosity parameter Ge. From Figure 2 it is also observed that (for gases Ge>0), for the values of Ge=+10, when the viscosity parameter variation is virtually negligible, we observed mixed convection profiles, typical of such standard convection flows, where as decreasing the values of Ge = 2 (i.e. increasing T_w-T_∞), effectively increasing the viscosity within the boundary layer, consistently reduces the velocity within the boundary layer. The variation of dimensionless temperature profiles G(η) are also presented in Figures 1 – 2 for Ge. From Figures 1 - 2 we conclude that the fluid temperature within the thermal boundary layer decreases as the viscosity parameter Ge increases. It is also observed that an increase in the values of Ge means a decrease in the temperature difference $\Delta T = T_w - T_\infty$.

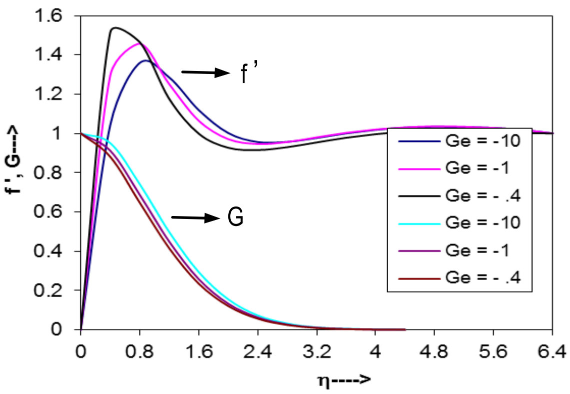

Figure 1

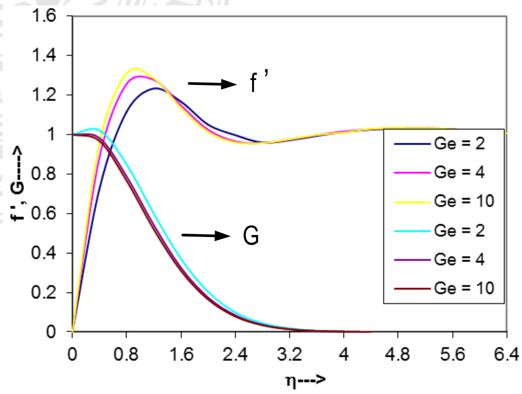

Figure 2

Velocity and temperature profiles with variation of Ge for Gr= -5 (figure 1) and Gr= 2 (figure 2) and for different values of the parameters Pr=.7, λ=-.5, λ_1=5, k=0,Ec=.1, α=10 and β=1.

It is observed from the Figures 3 - 4 that the velocity decreases with the dimensionless distance η from the flat surface with increasing values of the thermal conductivity parameter

Gr. Also the temperature within the thermal boundary layer first increases up to a certain point (say η=0.6 approximately) and then decreases as the thermal conductivity parameter Gr increases, which is observed from the Figures 3 - 4.

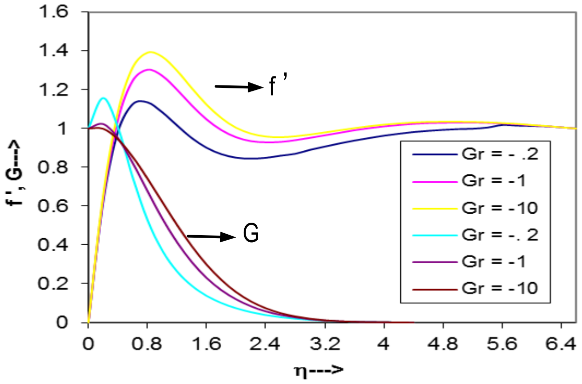

Figure 3

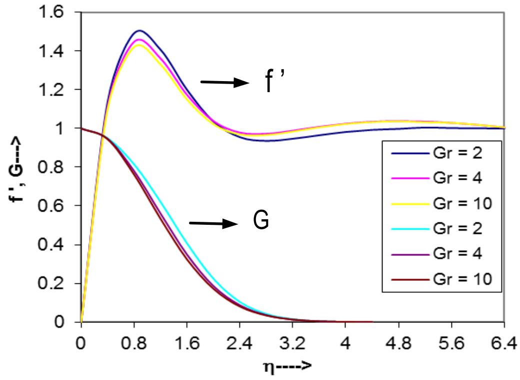

Figure 4

Velocity and temperature profiles with variation of Gr (figure 3-4) and for different values of the parameters Ge= -5, Pr=.7, Ec=.1, λ=-.5, λ_1=5, k=0, α=10 and β=1.

For prescribed wall temperature the effect of the Prandtl number Pr on the velocity and temperature profiles (f′, G) for λ = -0.5 (decelerating case), Ec =0, and different values of Ge and Gr are shown in Figures 5 – 6. Both velocity and thermal boundary layer thickness reduce as Pr increases. For Pr = 7, the temperature near the wall becomes more than the temperature at the wall. This implies that the hot wall will no longer be cooled and heat will be transferred from the fluid in the wall. It is observed that for Pr=7(water), there is no overshoot in the velocity profile. A high Prandtl number implies a more viscous fluid which retards the motion and thus suppress the overshoot in the velocity profile. It is also observed that the increasing values of Ge and Gr together with the increasing values of Pr, the velocity and thermal boundary layer thickness decreases.

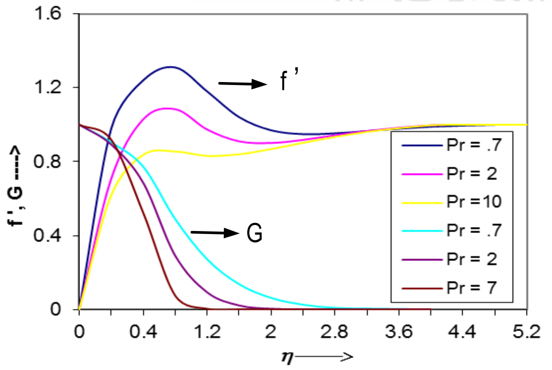

Figure 5

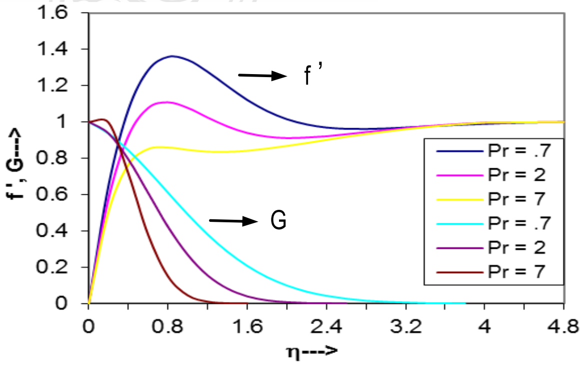

Figure 6

Velocity and temperature profiles with variation of Pr for Ge=Gr= -5 (figure 5) and Ge=Gr= 2 (figure 6) and for different values of the parameters, λ=-.5, λ_1=5, k=0=Ec, α=10 and β=.5.

4. Conclusion:

The results presented, demonstrate quite clearly that Ge and Gr, which are indicator of the variation of viscosity and thermal conductivity with temperature have a substantial effect on the drag and heat transfer characteristics as well as the velocity and temperature distributions within the boundary layer. Also the results indicate that the skin friction and heat transfer co-efficient and the induced magnetic field on the surface increase with buoyancy parameter which assists the forced flow. The buoyancy parameter causes overshoot in the velocity profile and it is further enhanced by the viscous dissipation.

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